The 8th Annual Vojtěch Jarník
International Mathematical Competition
Ostrava, 1st April 1998
Category I

Problem 1. Let a and d be two positive integers. Prove that there exists a constant K such that every set of K consecutive elements of the arithmetic progression $\{a+nd\}_{n=1}^{\infty}$ contains at least one number which is not prime. [15 points]

Problem 2. Find the limit

$$\lim_{n\to\infty}\left(\frac{\left(1+\frac{1}{n}\right)^n}{\mathrm{e}}\right)^n.$$

[20 points]

Problem 3. Give an example of a sequence of continuous functions on $\mathbb{R}$ converging pointwise to 0 which is not uniformly convergent on any nonempty open set. [30 points]

Problem 4-M. Prove the inequality

$$\frac{n\pi}{4}-\frac{1}{\sqrt{8n}}\leq\frac{1}{2}+\sum_{k=1}^{n-1}\sqrt{1-\frac{k^2}{n^2}}\leq\frac{n\pi}{4}$$

for every integer $n\geq 2$. [35 points]

Problem 4-I. Prove that there exists a program in standard Pascal which prints out its own ASCII code. No disk operations are permitted. [35 points]